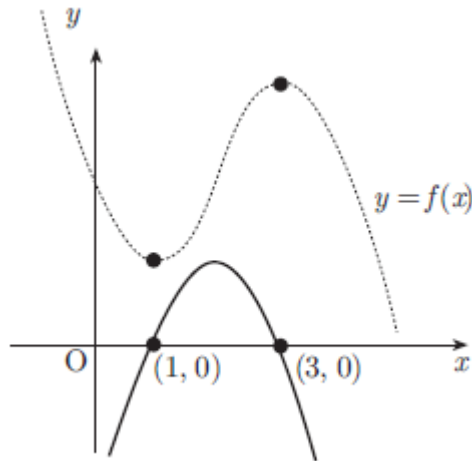


00 P1 A2	<p><b>4C, 2C, 1A</b></p> <ul style="list-style-type: none"> <li>•<sup>1</sup> <math>\frac{dy}{dx} = \dots\dots\dots</math></li> <li>•<sup>2</sup> <math>\frac{dy}{dx} = 3x^2 - 12x + 9</math></li> <li>•<sup>3</sup> <math>3x^2 - 12x + 9 = 0</math></li> <li>•<sup>4</sup> <math>A = (1, 4)</math></li> </ul> <p><math>f(x)</math> needs to be translated 4 units up, 2 units left</p> <ul style="list-style-type: none"> <li>•<sup>5</sup> sketch with coord of <math>A' (-1, 8)</math></li> <li>•<sup>6</sup> sketch with coord of <math>B' (1, 4)</math></li> <li>•<sup>7</sup> <math>4 &lt; k &lt; 8</math></li> </ul>
01 P1 Q9	<p><b>2C</b></p> <ul style="list-style-type: none"> <li>•<sup>1</sup> st line for <math>f'</math> thr' <math>(3, 0)</math>, <math>m_{f'} &gt; 0</math></li> <li>•<sup>2</sup> st line for <math>g'</math> thr' <math>(3, 0)</math>, <math>m_{f'} &gt; m_{g'} &gt; 0</math></li> </ul>
01 P2 Q2	<p><b>6C</b></p> <ul style="list-style-type: none"> <li>•<sup>1</sup> <math>(4, -4)</math> stated</li> <li>•<sup>2</sup> <math>-16x^{-\frac{1}{2}}</math></li> <li>•<sup>3</sup> <math>\frac{dy}{dx} = 1 \dots\dots\dots</math></li> <li>•<sup>4</sup> <math>\dots\dots\dots + 8x^{-\frac{3}{2}}</math></li> <li>•<sup>5</sup> <math>m_{x=4} = 2</math></li> <li>•<sup>6</sup> <math>y - (-4) = 2(x - 4)</math>      ans: <math>y = 2x - 12</math></li> </ul>
02 P1 Q4	<p><b>4C</b></p> <ul style="list-style-type: none"> <li>•<sup>1</sup> <math>\frac{dy}{dx} = 4x - 7</math></li> <li>•<sup>2</sup> <math>m_{\text{tang}} = \tan 45^\circ = 1</math></li> <li>•<sup>3</sup> <math>4x - 7 = 1</math></li> <li>•<sup>4</sup> <math>(2, 4)</math></li> </ul>
02 P1 Q6	<p><b>3C</b></p> <div style="display: flex; align-items: center;"> <div style="flex: 1;"> <ul style="list-style-type: none"> <li>•<sup>1</sup> roots at 0 and <math>c</math> (accept a statement to this effect)</li> <li>•<sup>2</sup> min at LH root, max between roots</li> <li>•<sup>3</sup> both 'tails' correct</li> </ul> </div> <div style="flex: 1; text-align: center;"> </div> </div>
6.(JA N) 02 P1	<p><b>5C</b></p>

	<ul style="list-style-type: none"> <li>•<sup>1</sup> <math>y = 3x^{-1}</math></li> <li>•<sup>2</sup> <math>\frac{dy}{dx} = -3x^{-2}</math></li> <li>•<sup>3</sup> <math>m_{x=1} = -3</math></li> <li>•<sup>4</sup> <math>y_{x=1} = 3</math></li> <li>•<sup>5</sup> <math>y - 3 = -3(x - 1)</math>      <b>ans:</b> <math>y + 3x = 6</math></li> </ul>
03 P1 Q5	<b>5C</b> <ul style="list-style-type: none"> <li>•<sup>1</sup> <math>x^{\frac{1}{2}}</math></li> <li>•<sup>2</sup> <math>2x^{-2}</math></li> <li>•<sup>3</sup> <math>\frac{1}{2}x^{-\frac{1}{2}}</math></li> <li>•<sup>4</sup> <math>-4x^{-3}</math></li> <li>•<sup>5</sup> <math>\frac{3}{16}</math></li> </ul>
04 P1 Q8	<b>2C, 4B</b> <ul style="list-style-type: none"> <li>•<sup>1</sup> <math>(x - 5)^2 \dots</math></li> <li>•<sup>2</sup> <math>(x - 5)^2 + 2</math></li> <li>•<sup>3</sup> <math>g'(x) =</math>      <i>STATED EXPLICITLY</i></li> <li>•<sup>4</sup> <math>x^2 - 10x + 27</math></li> <li>•<sup>5</sup> <math>(x - 5)^2 + 2</math></li> <li>•<sup>6</sup> <math>g'(x) &gt; 0</math> for all <math>x</math> and so <math>g(x)</math> increasing</li> </ul> <p><b>OR</b></p> <ul style="list-style-type: none"> <li>•<sup>3</sup> <math>g'(x) =</math>      <i>STATED EXPLICITLY</i></li> <li>•<sup>4</sup> <math>x^2 - 10x + 27</math></li> <li>•<sup>5</sup> <math>b^2 - 4ac = 100 - 108 = -8</math></li> <li>•<sup>6</sup> no roots, concave up, <math>g'(x) &gt; 0</math> and thus <math>g(x)</math> increasing</li> </ul>
04 P2 Q5	<b>5C, 2C</b> <ul style="list-style-type: none"> <li>•<sup>1</sup> <math>\frac{dy}{dx} =</math>      <i>stated</i></li> <li>•<sup>2</sup> <math>12x - 3x^2</math></li> <li>•<sup>3</sup> <math>12x - 3x^2 = 12</math></li> <li>•<sup>4</sup> <math>3(x - 2)^2 = 0</math></li> <li>•<sup>5</sup> <math>x = 2</math></li> <li>•<sup>6</sup> <math>y = 16</math></li> <li>•<sup>7</sup> <math>y - 16 = 12(x - 2)</math></li> </ul>
04 P2 Q7	<b>3B</b>

a sketch with the following details

- <sup>1</sup> only two intercepts on the  $x$  - axis at 1 and 3
- <sup>2</sup> function is +ve between the roots and - ve outwith
- <sup>3</sup> a parabola (symmetrical about midpoint of  $x$  - intercepts), stated or implied by the accuracy of the diagram



05  
P2  
Q6

**6B**

- <sup>1</sup>  $\frac{dy}{dx} = \dots$
- <sup>2</sup>  $y = 24x^{-\frac{1}{2}}$
- <sup>3</sup>  $\frac{dy}{dx} = -12x^{-\frac{3}{2}}$
- <sup>4</sup>  $\frac{dy}{dx} \bigg|_{x=4} = -\frac{3}{2}$
- <sup>5</sup>  $y_{x=4} = 12$
- <sup>6</sup>  $y - 12 = -\frac{3}{2}(x - 4)$        $2y + 3x = 36$

07  
P1  
Q9

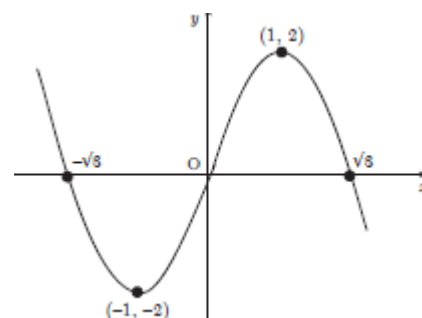
**2C/B, 7C/B, 1B**

- <sup>1</sup> any two of  $x = 0$ ,  $x = \sqrt{3}$  and  $x = -\sqrt{3}$
- <sup>2</sup> remaining one
- <sup>3</sup>  $f'(x) =$
- <sup>4</sup>  $3 - 3x^2$
- <sup>5</sup>  $f'(x) = 0$

$$\begin{array}{c|c|c} \bullet^6 & \bullet^7 & \\ \hline x & 1 & -1 \\ \hline \bullet^7 & y & 2 \end{array} \quad \begin{array}{c} -2 \\ \text{s/i by the sketch} \end{array}$$

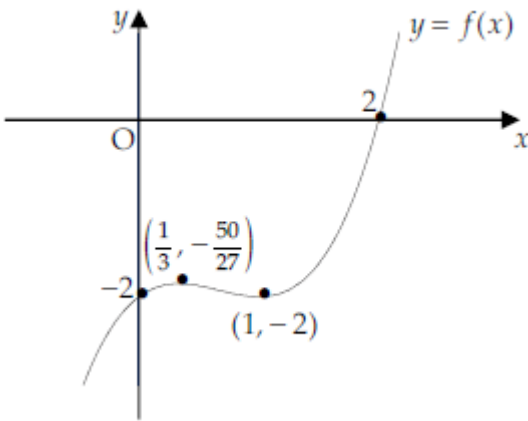
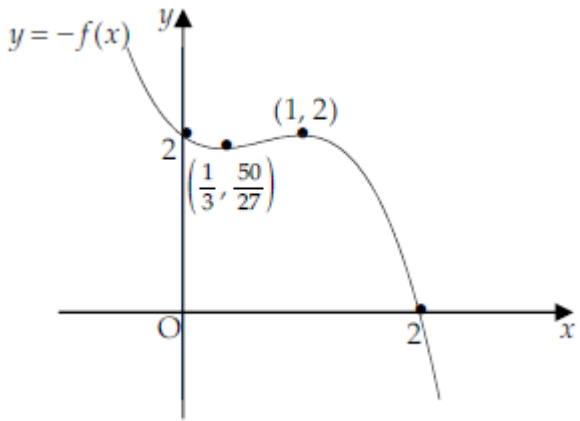
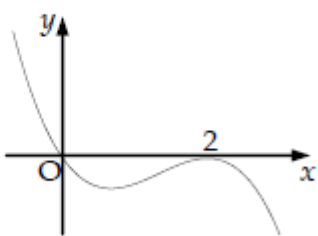
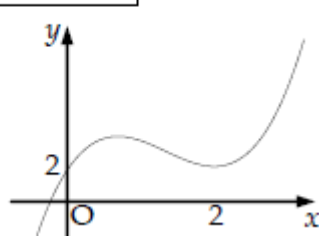
		$\bullet^8$		$\bullet^9$	
$\bullet^8$	...	-1	...	1	...
$\bullet^9$	$f'$	-	0	+	+
		minimum		maximum	

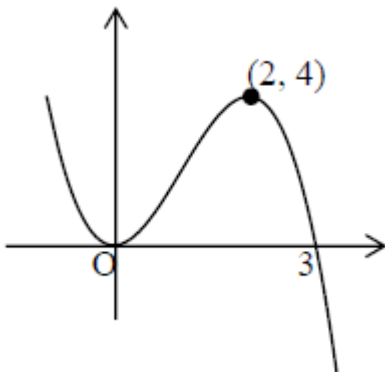
- <sup>10</sup> sketch(see below)



07 P2 Q5	<p><b>5C, 2B, 2B</b></p> <p>•<sup>1</sup> <math>\frac{dy}{dx} = \dots</math> (1 term correct)</p> <p>•<sup>2</sup> <math>x - 8</math></p> <p>•<sup>3</sup> <math>x - 8 = 4</math></p> <p>•<sup>4</sup> <math>x = 12</math></p> <p>•<sup>5</sup> <math>y = 10</math></p> <p>•<sup>6</sup> <math>m_P = -4</math></p> <p>•<sup>7</sup> <math>P = (4, 10)</math></p> <p>•<sup>8</sup> <math>x_O = 8</math></p> <p>•<sup>9</sup> <math>y_O = 11</math></p>	<p><i>Alternative Method for (c)</i></p> <p>Solving the normals</p> <p>i.e. <math>y - 10 = -\frac{1}{4}(x - 12)</math></p> <p><math>y - 10 = \frac{1}{4}(x - 4)</math></p> <p>may be used. Marks are awarded as normal:</p> <p><math>x = 8</math> (•<sup>8</sup>) and <math>y = 11</math> (•<sup>9</sup>)</p>
08 P1 Q22	<p><b>5C, 2C</b></p> <p>•<sup>1</sup> <math>\frac{dy}{dx} = \dots</math> (1 term correct) s / i by •<sup>2</sup></p> <p>•<sup>2</sup> <math>3x^2 - 12x + 8</math> s / i by •<sup>3</sup></p> <p>•<sup>3</sup> <math>3x^2 - 12x + 8 = -1</math></p>	<p><b>Alternative for •<sup>6</sup> and •<sup>7</sup></b></p> <p>•<sup>4</sup> <math>x</math>   •<sup>4</sup> 1   •<sup>5</sup> 3  </p> <p>•<sup>5</sup> <math>y</math>   3   -3  </p> <p>•<sup>6</sup> <math>y = 4 - x</math> has gradient <math>= -1</math></p> <p>•<sup>7</sup> check (3, -3) and reject</p> <p>check (1, 3) and accept</p> <p>•<sup>6</sup> <math>\begin{cases} x^3 - 6x^2 + 8x = 4 - x \\ x^3 - 6x^2 + 9x - 4 = 0 \\ (x - 1)(x^2 - 5x + 4) \\ (x - 4)(x - 1) \end{cases}</math></p> <p>•<sup>7</sup> <math>\begin{cases} \text{repeated root implies} \\ \text{tangent at (1, 3).} \end{cases}</math></p>
09 P2 Q1	<b>8C</b>	



	<ul style="list-style-type: none"> <li>•<sup>11</sup> ic curve showing points from (a) and (b) without annotation</li> <li>•<sup>12</sup> ic <b>cubic</b> curve showing <b>all</b> intercepts and stationary points annotated</li> <li>•<sup>13</sup> ic curve from (i) reflected in <math>x</math>-axis</li> </ul> <div style="display: flex; justify-content: space-around;">   </div>
12 P2 Q3	<div style="display: flex;"> <div style="flex: 1;"> <p>start to differentiate</p> <p>complete derivative and set to 0</p> <p>start to solve <math>f'(x) = 0</math></p> <p>solve <math>f'(x) = 0</math></p> <p>evaluate <math>f</math> at relevant stationary point</p> <p>consider end-points</p> <p>state max. and min. values</p> </div> <div style="flex: 1;"> <ul style="list-style-type: none"> <li>•<sup>1</sup> differentiate <math>x^3</math> or <math>-2x^2</math> correctly</li> <li>•<sup>2</sup> <math>3x^2 - 4x - 4</math></li> <li>•<sup>3</sup> e.g. <math>(3x+2)(x-2)</math> } <math>= 0</math> must appear at •<sup>2</sup>.</li> <li>•<sup>4</sup> <math>-\frac{2}{3}, 2</math></li> <li>•<sup>5</sup> <math>f(2) = -2</math></li> <li>•<sup>6</sup> <math>f(0) = 6</math> and <math>f(3) = 3</math></li> <li>•<sup>7</sup> max. 6 and min. -2</li> </ul> </div> </div>
12 P2 Q4	<div style="display: flex;"> <div style="flex: 1;"> <p>identify roots</p> <p>interpret point of inflection</p> <p>complete cubic curve</p> <p>reflection in <math>x</math>-axis</p> <p>translation <math>\begin{bmatrix} 0 \\ 2 \end{bmatrix}</math></p> <p>annotation of 'transformed' graph</p> </div> <div style="flex: 1;"> <ul style="list-style-type: none"> <li>•<sup>1</sup> 0 and 2 only</li> <li>•<sup>2</sup> turning point at (2, 0)</li> <li>•<sup>3</sup> cubic, passing through O with negative gradient</li> <li>•<sup>4</sup> reflection of graph in (a) in <math>x</math>-axis</li> <li>•<sup>5</sup> graph moves parallel to <math>y</math>-axis by 2 units upwards</li> <li>•<sup>6</sup> two 'transformed' points appropriately annotated</li> </ul> </div> </div> <div style="display: flex; justify-content: space-around; margin-top: 10px;"> <div style="border: 1px solid black; padding: 5px; text-align: center;">Graph for (a)</div> <div style="border: 1px solid black; padding: 5px; text-align: center;">Graph for (b)</div> </div> <div style="display: flex; justify-content: space-around; margin-top: 10px;">   </div>
14 P1 Q21	<div style="display: flex;"> <div style="flex: 1;"> <p>know to differentiate and one term correct</p> <p>the other term correct and set derivative to 0</p> </div> <div style="flex: 1;"> <ul style="list-style-type: none"> <li>•<sup>1</sup> <math>= 6x \dots</math> or <math>= \dots - 3x^2</math></li> <li>•<sup>2</sup> <math>6x - 3x^2 = 0</math> stated explicitly</li> </ul> </div> </div>

	<p>solve <math>\frac{dy}{dx} = 0</math></p> <p>evaluate <math>y</math> coordinates</p> <p>justify nature of stationary points</p> <p>interpretation</p> <p>find intercepts</p> <p>sketch</p>	<div style="display: flex; align-items: center;"> <div style="margin-right: 10px;"> <math>\bullet^3</math>  <math>\bullet^4</math>  <math>\bullet^5</math>  <math>\bullet^6</math> </div> <div> <math>x = 0</math>  <math>y = 0</math>              use 2<sup>nd</sup> derivative or nature table              min. at (0,0) and max. at (2,4)           </div> </div> <table style="margin-left: auto; margin-right: auto; border-collapse: collapse;"> <tr> <td></td><td style="border-right: 1px solid black; padding: 5px;">0</td><td style="padding: 5px;">2</td></tr> <tr> <td style="border-right: 1px solid black; padding: 5px;"><math>\frac{dy}{dx}</math></td><td style="padding: 5px;">- 0 + 0 -</td><td></td></tr> </table> <div style="text-align: center; margin-top: 10px;">  </div> <div style="margin-top: 20px;"> <math>\bullet^7</math> <math>3x^2 - x^3 = 0</math> and (3,0) or <math>x = 3</math>              (0,0) [may appear in part a]           </div> <div style="margin-top: 10px;"> <math>\bullet^8</math> sketch           </div>		0	2	$\frac{dy}{dx}$	- 0 + 0 -	
	0	2						
$\frac{dy}{dx}$	- 0 + 0 -							
14 P2 Q2	<p>know to and differentiate</p> <p>find gradient</p> <p>find <math>y</math>-coordinate</p> <p>state equation of tangent</p>	<div style="display: flex; align-items: center;"> <div style="margin-right: 10px;"> <math>\bullet^1</math>  <math>\bullet^2</math>  <math>\bullet^3</math>  <math>\bullet^4</math> </div> <div> <math>4x^3 - 6x^2</math>              8              5  <math>y - 5 = 8(x - 2)</math> </div> </div>						

### ANSWERS Pre 2000 – Basic Differentiation

1	<p>•<sup>1</sup> know to expand</p> <p>•<sup>2</sup> <math>2x^{\frac{3}{2}} + 4x^{\frac{1}{2}}</math></p> <p>•<sup>3</sup> <math>3x^{\frac{1}{2}}</math></p> <p>•<sup>4</sup> <math>2x^{-\frac{1}{2}}</math></p>
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2	<ul style="list-style-type: none"><li>•<sup>1</sup> <math>4x^{-2}</math> stated or implied by •<sup>3</sup></li><li>•<sup>2</sup> <math>+x^{\frac{3}{2}}</math> stated or implied by •<sup>4</sup></li><li>•<sup>3</sup> <math>-8x^{-3}</math></li><li>•<sup>4</sup> <math>+\frac{3}{2}x^{\frac{1}{2}}</math></li></ul>						
3	<ul style="list-style-type: none"><li>•<sup>1</sup> <math>\frac{dy}{dx} = 4x + 1</math></li><li>•<sup>2</sup> <math>LHS = x(1 + 4x + 1)</math> or <math>RHS = 2(2x^2 + x)</math></li><li>•<sup>3</sup> completes proof</li></ul>						
4	<ul style="list-style-type: none"><li>•<sup>1</sup> <math>\frac{dy}{dx} = 2x - 1</math></li><li>•<sup>2</sup> <math>RHS = 1 + \frac{2(x^2 - x)}{x}</math></li><li>•<sup>3</sup> <math>1 + 2(x - 1)</math> and complete</li></ul>						
5	<ul style="list-style-type: none"><li>•<sup>1</sup> <math>6x^3 - 3x^2</math></li><li>•<sup>2</sup> <math>18x^2 - 6x</math></li><li>•<sup>3</sup> 24</li></ul>						
6	<div><div><ul style="list-style-type: none"><li>•<sup>1</sup> <math>\frac{x}{\sqrt{x}} - \frac{1}{\sqrt{x}}</math> or <math>x \times x^{-\frac{1}{2}} - 1 \times x^{-\frac{1}{2}}</math></li><li>•<sup>2</sup> <math>x^{\frac{1}{2}} - x^{-\frac{1}{2}}</math></li></ul></div><div><ul style="list-style-type: none"><li>•<sup>3</sup> <math>\frac{1}{2}x^{-\frac{1}{2}}</math></li><li>•<sup>4</sup> <math>\frac{1}{2}x^{-\frac{3}{2}}</math></li><li>•<sup>5</sup> <math>\frac{5}{16}</math></li></ul></div></div>						
7	<ul style="list-style-type: none"><li>•<sup>1</sup> <math>f'(x) = 3kx^2 + 5</math></li><li>•<sup>2</sup> <math>f'(1) = 3k + 5</math></li><li>•<sup>3</sup> <math>k = 3</math></li></ul>						
8	<div>3C, 3C</div> <div><div><ul style="list-style-type: none"><li>•<sup>1</sup> <math>f(x^2 - 2)</math></li><li>•<sup>2</sup> <math>3(x^2 - 2) + 1</math></li><li>•<sup>3</sup> <math>(3x + 1)^2 - 2</math></li></ul></div><div><table><tr><td>•<sup>4</sup></td><td>•<sup>5</sup></td></tr><tr><td><math>3x^2 - 5</math></td><td><math>9x^2 + 6x - 1</math></td></tr><tr><td><math>6x</math></td><td><math>18x + 6</math> or equiv.</td></tr></table></div></div> <div><div><math>x = -\frac{1}{2}</math></div><div>s / l by •<sup>2</sup></div><div>s / l by •<sup>5</sup></div></div>	• <sup>4</sup>	• <sup>5</sup>	$3x^2 - 5$	$9x^2 + 6x - 1$	$6x$	$18x + 6$ or equiv.
• <sup>4</sup>	• <sup>5</sup>						
$3x^2 - 5$	$9x^2 + 6x - 1$						
$6x$	$18x + 6$ or equiv.						
9	<ul style="list-style-type: none"><li>•<sup>1</sup> gradient = <math>-\frac{1}{3}</math></li><li>•<sup>2</sup> <math>f'(x) = \text{gradient}</math></li></ul>						
10	<div><div><div><ul style="list-style-type: none"><li>•<sup>1</sup> <math>\frac{dh}{dt} = \dots\dots</math></li><li>•<sup>2</sup> <math>19.6 - 9.8t</math></li><li>•<sup>3</sup> 9.8</li></ul></div><div><ul style="list-style-type: none"><li>•<sup>4</sup> <math>\frac{dh}{dt} = 0</math></li><li>•<sup>5</sup> <math>t = 2</math></li></ul></div><div>Alternative<ul style="list-style-type: none"><li>•<sup>4</sup> <math>h(t)</math> is a parabola which is <b>symmetric</b> about its <b>maximum</b></li><li>•<sup>5</sup> (e.g.) <math>h(1) = 15.9</math>, <math>h(2) = 20.8</math>, <math>h(3) = 15.9</math> so <math>t = 2</math></li></ul></div></div></div>						



11	<ul style="list-style-type: none"> <li>•<sup>1</sup> knows to differentiate</li> <li>•<sup>2</sup> <math>20 - 10t</math></li> <li>•<sup>3</sup> 20</li> <li>•<sup>4</sup> speed = 0</li> <li>•<sup>5</sup> ball stationary at top of flight</li> </ul>
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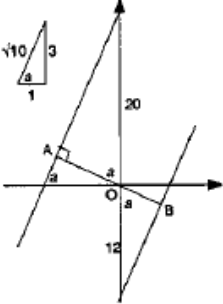
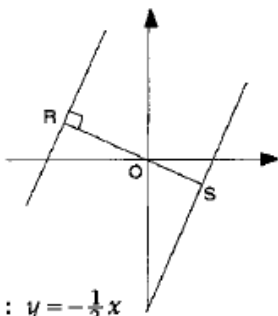
### ANSWERS Pre 2000 – Equation of Tangent

1	<ul style="list-style-type: none"> <li>•<sup>1</sup> <math>y' = 15x^2 - 12x</math></li> <li>•<sup>2</sup> <math>y'(1) = 3</math></li> <li>•<sup>3</sup> <math>y(1) = -1</math></li> <li>•<sup>4</sup> <math>y - (-1) = 3(x - 1)</math></li> </ul>
2	<ul style="list-style-type: none"> <li>•<sup>1</sup> strat: <math>\frac{dy}{dx} = \dots\dots</math></li> <li>•<sup>2</sup> <math>f'(1) = 6</math></li> <li>•<sup>3</sup> <math>f(1) = 5</math></li> <li>•<sup>4</sup> <math>y - 5 = 6(x - 1)</math></li> </ul>
3	<ul style="list-style-type: none"> <li>•<sup>1</sup> strat: <math>\frac{dy}{dx} = \dots\dots</math></li> <li>•<sup>2</sup> <math>\frac{dy}{dx} = 12x^2</math></li> <li>•<sup>3</sup> <math>m = 12</math></li> <li>•<sup>4</sup> <math>y - (-6) = 12(x - (-1))</math></li> </ul>
4	<ul style="list-style-type: none"> <li>•<sup>1</sup> <math>\frac{dy}{dx} = \dots\dots</math></li> <li>•<sup>2</sup> <math>10x</math></li> <li>•<sup>3</sup> <math>-10</math></li> <li>•<sup>4</sup> <math>y - 7 = -10(x - (-1))</math></li> </ul>
5	<p>(a)</p> <ul style="list-style-type: none"> <li>•<sup>1</sup> <math>\frac{dy}{dx} = \dots</math></li> <li>•<sup>2</sup> <math>3x^2 - 8x + 2</math></li> <li>•<sup>3</sup> gradient = <math>-2</math> (calculated from <math>\frac{dy}{dx}</math>)</li> <li>•<sup>4</sup> <math>y_A = -5</math></li> <li>•<sup>5</sup> <math>y + 5 = -2(x - 2)</math></li> </ul> <p>(b)</p> <ul style="list-style-type: none"> <li>•<sup>6</sup> <math>m_{\text{normal}} = \frac{1}{2}</math></li> <li>•<sup>7</sup> angle = <math>\tan^{-1} \frac{1}{2}</math></li> </ul>

11	<p>(a)</p> <ul style="list-style-type: none"> <li>•<sup>1</sup> <math>\frac{1}{x} = x^{-1}</math></li> <li>•<sup>2</sup> <math>\frac{dy}{dx} = \dots\dots</math></li> <li>•<sup>3</sup> <math>\frac{dy}{dx} = -x^{-2}</math></li> <li>•<sup>4</sup> gradient = <math>-a^{-2}</math></li> </ul> <p>(b)</p> <ul style="list-style-type: none"> <li>•<sup>5</sup> use <math>y - \frac{1}{a} = -\frac{1}{a^2}(x - a)</math></li> <li>•<sup>6</sup> <math>a^2y - a = -(x - a)</math> and completes proof</li> </ul> <p>(c)</p> <ul style="list-style-type: none"> <li>•<sup>7</sup> <math>y_B = \frac{2a}{a^2}</math></li> <li>•<sup>8</sup> <math>x_A = 2a</math></li> <li>•<sup>9</sup> 2</li> <li>•<sup>10</sup> independent of <math>a</math></li> </ul>
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### ANSWERS Pre 2000 Questions – Tangent Problems

1	<ul style="list-style-type: none"> <li>•<sup>1</sup> <math>\frac{dy}{dx} = \dots\dots</math></li> <li>•<sup>2</sup> <math>6x^2 - 6x - 12</math></li> <li>•<sup>3</sup> <math>\dots\dots = 0</math></li> <li>•<sup>4</sup> <math>x = -1, 2</math></li> </ul>						
2	<ul style="list-style-type: none"> <li>•<sup>1</sup> <math>\frac{dy}{dx} = \dots\dots</math></li> <li>•<sup>2</sup> <math>3x^2 + 2kx - 8</math></li> <li>•<sup>3</sup> <math>3x^2 + 2kx - 8 = 0</math> when <math>x = -2</math></li> <li>•<sup>4</sup> <math>k = 1</math></li> </ul>						
3	<ul style="list-style-type: none"> <li>•<sup>1</sup> equate gradients</li> <li>•<sup>2</sup> <math>m = -5</math></li> <li>•<sup>3</sup> <math>\frac{dy}{dx} = \dots</math></li> <li>•<sup>4</sup> <math>\frac{dy}{dx} = 8x + 3</math></li> <li>•<sup>5</sup> <math>x = -1</math></li> </ul>						
4	<ul style="list-style-type: none"> <li>•<sup>1</sup> <math>\frac{dy}{dx} = 3x^2 - 4</math></li> <li>•<sup>2</sup> <math>\frac{dy}{dx}_{x=2} = 8</math></li> <li>•<sup>3</sup> <math>\tan \theta = 8</math></li> <li>•<sup>4</sup> <math>83^\circ</math></li> </ul>						
5	<table border="0"> <tr> <td>•<sup>1</sup> know to differentiate</td><td>•<sup>4</sup> <math>76^\circ</math></td></tr> <tr> <td>•<sup>2</sup> <math>4 - 2x</math></td><td>•<sup>5</sup> <math>45^\circ</math></td></tr> <tr> <td>•<sup>3</sup> <math>m = 4</math></td><td>•<sup>6</sup> <math>31^\circ</math></td></tr> </table>	• <sup>1</sup> know to differentiate	• <sup>4</sup> $76^\circ$	• <sup>2</sup> $4 - 2x$	• <sup>5</sup> $45^\circ$	• <sup>3</sup> $m = 4$	• <sup>6</sup> $31^\circ$
• <sup>1</sup> know to differentiate	• <sup>4</sup> $76^\circ$						
• <sup>2</sup> $4 - 2x$	• <sup>5</sup> $45^\circ$						
• <sup>3</sup> $m = 4$	• <sup>6</sup> $31^\circ$						

6	<p>(a)</p> <ul style="list-style-type: none"> <li>•<sup>1</sup> strategy: <math>\frac{dy}{dx} = \dots = 3</math></li> <li>•<sup>2</sup> <math>3x^2 - 9</math></li> <li>•<sup>3</sup> <math>x = 2, -2</math> <b>OR</b> <span style="border: 1px solid black; padding: 2px; display: inline-block;"> <ul style="list-style-type: none"> <li>•<sup>3</sup> <math>x = 2, y = -6</math></li> <li>•<sup>4</sup> <math>x = -2, y = 14</math></li> </ul> </span></li> <li>•<sup>4</sup> <math>y = -6, 14</math></li> <li>•<sup>5</sup> <math>y + 6 = 3(x - 2)</math></li> <li>•<sup>6</sup> <math>y - 14 = 3(x + 2)</math></li> </ul> <p>(b)</p> <ul style="list-style-type: none"> <li>•<sup>7</sup> diagram with <math>y = -\frac{1}{3}x</math></li> <li>•<sup>8</sup> for 20 and 12</li> <li>•<sup>9</sup> <math>AB = AO + OB</math></li> <li>•<sup>10</sup> <math>AB = 20 \cos a + 12 \cos a</math></li> <li>•<sup>11</sup> using <math>\tan a = \frac{3}{4}</math></li> <li>•<sup>12</sup> <math>AB = 32 \times \frac{1}{\sqrt{10}} = 32 \times \frac{\sqrt{10}}{10} = \frac{32}{5} \sqrt{10}</math></li> </ul>   <ul style="list-style-type: none"> <li>•<sup>7</sup> <math>m_{RS} = -\frac{1}{3}</math></li> <li>•<sup>8</sup> equ of RS : <math>y = -\frac{1}{3}x</math></li> <li>•<sup>9</sup> <math>-\frac{1}{3}x = 3x - 12</math> &amp; <math>-\frac{1}{3}x = 3x + 20</math></li> <li>•<sup>10</sup> <math>R(-6, 2)</math> and <math>S(\frac{18}{5}, -\frac{6}{5})</math></li> <li>•<sup>11</sup> <math>d^2 = \left(-6 - \left(\frac{18}{5}\right)\right)^2 + \left(2 - \left(-\frac{6}{5}\right)\right)^2</math></li> <li>•<sup>12</sup> <math>d^2 = \frac{48^2}{25} + \frac{16^2}{25}</math> and completes proof</li> </ul>
7	<ul style="list-style-type: none"> <li>•<sup>1</sup> <math>c = 3</math></li> <li>•<sup>2</sup> <math>2ax + b</math></li> <li>•<sup>3</sup> <math>m = \tan 45^\circ = 1</math></li> <li>•<sup>4</sup> <math>-2a + b = 1</math></li> <li>•<sup>5</sup> <math>m = \tan 135^\circ = -1</math></li> <li>•<sup>6</sup> <math>4a + b = -1</math></li> <li>•<sup>7</sup> method for solving pr. of equ</li> <li>•<sup>8</sup> <math>a = -\frac{1}{3}, b = \frac{1}{3}</math></li> </ul>

### ANSWERS Pre 2000 – Stationary Points

1

- <sup>1</sup> for knowing to differentiate
- <sup>2</sup>  $f'(x) = 4x^3 - 6x^2 + 2$
- <sup>3</sup> for putting  $f'(x) = 0$
- <sup>4</sup> for factorising or checking zeros
- <sup>5</sup>  $x = -\frac{1}{2}, x = 1$
- <sup>6</sup>  $y = -\frac{27}{16}, y = 0$
- <sup>7</sup> completed nature table

$x$	$< -\frac{1}{2}$	$-\frac{1}{2}$	$> -\frac{1}{2}$	$< 1$	$1$	$> 1$
$f'(x)$	$-ve$	$0$	$+ve$	$+ve$	$0$	$+ve$
	$\searrow$	$\_\_\_\_\_\_$	$\nearrow$	$\nearrow$	$\_\_\_\_\_\_$	$\nearrow$
- <sup>8</sup>  $(1, 0)$  is pt. of inflexion,  $(-\frac{1}{2}, -1\frac{11}{16})$  is min t.p.

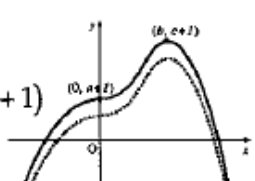
2	<p>(a)</p> <ul style="list-style-type: none"> <li><math>\frac{dy}{dx} = \dots\dots</math> <i>stated or implied by</i></li> <li><math>-4x^3 + 12x^2</math></li> <li><math>-4x^3 + 12x^2 = 0</math> or <math>\frac{dy}{dx} = 0</math> <i>explicitly stated</i></li> <li><math>-4x^2(x-3)</math> (accept <math>x^2(-4x+12)</math>)</li> <li><math>x=0</math> and <math>3</math></li> <li><math>y=-2</math> and <math>25</math></li> </ul> <p>(b)</p> <table border="1"> <tr> <td><math>x</math></td><td><math>0^-</math></td><td><math>0</math></td><td><math>0^+</math></td><td><math>3</math></td><td><math>3^+</math></td></tr> <tr> <td><math>\frac{dy}{dx}</math></td><td></td><td></td><td></td><td></td><td></td></tr> </table> <ul style="list-style-type: none"> <li> <table> <tr> <td></td><td>+</td><td>0</td><td>+</td><td>0</td><td>-</td></tr> <tr> <td>PI at <math>x=0</math>,</td><td></td><td></td><td>max at <math>x=3</math></td><td></td><td></td></tr> </table> </li> </ul>	$x$	$0^-$	$0$	$0^+$	$3$	$3^+$	$\frac{dy}{dx}$							+	0	+	0	-	PI at $x=0$ ,			max at $x=3$		
$x$	$0^-$	$0$	$0^+$	$3$	$3^+$																				
$\frac{dy}{dx}$																									
	+	0	+	0	-																				
PI at $x=0$ ,			max at $x=3$																						
3	<p>(a)</p> <ul style="list-style-type: none"> <li><math>\frac{dy}{dx} =</math></li> <li><math>4x^3 - 12x^2</math></li> <li><math>= 0</math> <i>stated explicitly</i></li> <li>e.g. <math>4x^2(x-3)</math></li> <li><math>x=0, 3</math></li> <li><math>y=3, -24</math></li> </ul> <p>(b)</p> <ul style="list-style-type: none"> <li> <table> <tr> <td><math>x</math></td><td><math>0^-</math></td><td><math>0</math></td><td><math>0^+</math></td><td><math>3</math></td><td><math>3^+</math></td></tr> <tr> <td><math>\frac{dy}{dx}</math></td><td>-</td><td>0</td><td>-</td><td>0</td><td>+</td></tr> </table> </li> <li>pt of inflection at <math>x=0</math> minimum at <math>x=3</math></li> </ul>	$x$	$0^-$	$0$	$0^+$	$3$	$3^+$	$\frac{dy}{dx}$	-	0	-	0	+												
$x$	$0^-$	$0$	$0^+$	$3$	$3^+$																				
$\frac{dy}{dx}$	-	0	-	0	+																				
4	<p>(a)</p> <ul style="list-style-type: none"> <li><math>\frac{dy}{dx} = 6x^2 - 12x</math></li> <li><math>\frac{dy}{dx} = 0</math></li> <li><math>x=0, x=2</math></li> <li> <table> <tr> <td><math>x</math></td><td><math>0^-</math></td><td><math>0</math></td><td><math>0^+</math></td><td><math>2^-</math></td><td><math>2</math></td><td><math>2^+</math></td></tr> <tr> <td><math>\frac{dy}{dx}</math></td><td>+</td><td>-</td><td>+</td><td>+</td><td>-</td><td>+</td></tr> </table> </li> <li>max. at <math>(0,0)</math> min at <math>(2,-8)</math></li> </ul> <p>(b)</p> <ul style="list-style-type: none"> <li><math>k &lt; 0</math></li> <li><math>k &gt; -8</math></li> </ul>	$x$	$0^-$	$0$	$0^+$	$2^-$	$2$	$2^+$	$\frac{dy}{dx}$	+	-	+	+	-	+										
$x$	$0^-$	$0$	$0^+$	$2^-$	$2$	$2^+$																			
$\frac{dy}{dx}$	+	-	+	+	-	+																			

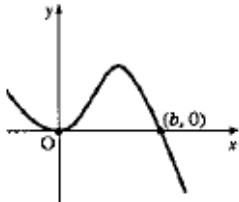
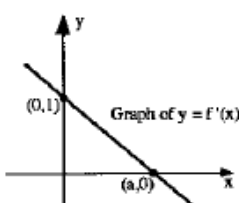
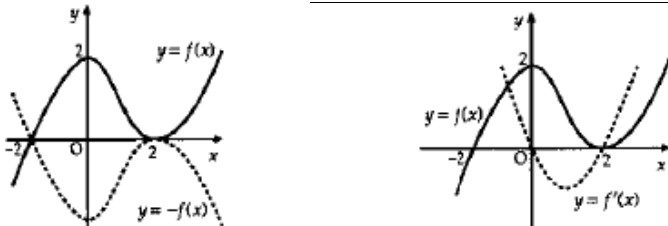

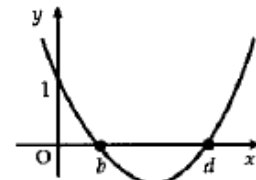
5	<p>(a)</p> <ul style="list-style-type: none"><li>•<sup>1</sup> <math>x = 1, -2</math></li><li>•<sup>2</sup> <math>(1,0)</math> and <math>(-2,0)</math></li><li>•<sup>3</sup> <math>(0,2)</math></li></ul> <p>(b)</p> <ul style="list-style-type: none"><li>•<sup>4</sup> <math>f(x) = x^3 - 3x + 2</math></li><li>•<sup>5</sup> <math>f'(x) = 3x^2 - 3</math></li><li>•<sup>6</sup> <math>f'(x) = 0</math> <b>stated explicitly</b></li><li>•<sup>7</sup> <math>x = 1</math> and <math>-1</math></li><li>•<sup>8</sup><table style="display: inline-table; border: none;"><tr><td><math>x</math></td><td><math>-1^-</math></td><td><math>-1</math></td><td><math>-1^+</math></td><td><math>1^-</math></td><td><math>1</math></td><td><math>1^+</math></td></tr><tr><td><math>f'(x)</math></td><td><math>+</math></td><td><math>0</math></td><td><math>-</math></td><td><math>-</math></td><td><math>0</math></td><td><math>+</math></td></tr></table></li><li>•<sup>9</sup> max at <math>(-1,4)</math></li><li>•<sup>10</sup> min at <math>(1,0)</math></li></ul> <p>(c)</p> <ul style="list-style-type: none"><li>•<sup>11</sup> correct shape of sketch</li><li>•<sup>12</sup> correct annotation of sketch(max,min, 2 axes intersections)</li></ul>	$x$	$-1^-$	$-1$	$-1^+$	$1^-$	$1$	$1^+$	$f'(x)$	$+$	$0$	$-$	$-$	$0$	$+$
$x$	$-1^-$	$-1$	$-1^+$	$1^-$	$1$	$1^+$									
$f'(x)$	$+$	$0$	$-$	$-$	$0$	$+$									

### ANSWERS Pre 2000 – Increasing/Decreasing Functions

1	<ul style="list-style-type: none"> <li>•<sup>1</sup> know to consider <math>f'(x) &gt; 0</math>      stated or implied by the evidence for •<sup>4</sup>.</li> <li>•<sup>2</sup> <math>\frac{dy}{dx} = 6x^2 - 6x - 36</math></li> <li>•<sup>3</sup> <math>6(x-3)(x+2) &gt; 0</math>      or by formula or completing the square</li> <li>•<sup>4</sup> <math>x &lt; -2, x &gt; 3</math></li> </ul>
2	<ul style="list-style-type: none"> <li>•<sup>1</sup> <math>b = -10</math></li> <li>•<sup>2</sup> know to differentiate and know to show <math>\frac{dy}{dx} \big _{x=-2} &gt; 0</math></li> <li>•<sup>3</sup> <math>9x^2 - 2x - 7</math></li> <li>•<sup>4</sup> show that <math>\frac{dy}{dx} \big _{x=-2} &gt; 0</math></li> </ul>
3	<ul style="list-style-type: none"> <li>•<sup>1</sup> <math>f'(x) = x^2 - 4x - 5</math></li> <li>•<sup>2</sup> use <math>f'(x) &gt; 0</math></li> <li>•<sup>3</sup> zeros at <math>x = 5</math> and <math>x = -1</math></li> <li>•<sup>4</sup> strat. e.g. for <math>-1 &lt; x &lt; 5</math> test <math>x = 0</math></li> <li>•<sup>5</sup> <math>x &lt; -1, x &gt; 5</math></li> </ul>

### ANSWERS Pre 2000 – Graphs of $y = f'(x)$

1	<ul style="list-style-type: none"> <li>•<sup>1</sup> translation <math>\begin{pmatrix} 0 \\ 1 \end{pmatrix}</math></li> <li>•<sup>2</sup> annotate <math>(0, a+1)</math> &amp; <math>(b, c+1)</math></li> </ul> 
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	<ul style="list-style-type: none"> <li>•<sup>3</sup> roots at <math>(0,0)</math> &amp; <math>(b,0)</math></li> <li>•<sup>4</sup> <math>y' &gt; 0</math> for <math>x &lt; b</math>, <math>y' &lt; 0</math> for <math>x &gt; b</math></li> <li>•<sup>5</sup> they coincide</li> </ul> 
2	<ul style="list-style-type: none"> <li>•<sup>1</sup> <math>(-2,0)</math> and <math>(3,0)</math></li> <li>•<sup>2</sup> know how to find sign of <math>f'</math> over 3 intervals</li> <li>•<sup>3</sup> min tp between <math>x = -2</math> and <math>x = 3</math> and no other</li> </ul>
3	<ul style="list-style-type: none"> <li>•<sup>1</sup> <math>f'(a) = 0</math></li> <li>•<sup>2</sup> <math>m_{\text{tgt}}</math> at <math>(0,0) = 1</math></li> <li>•<sup>3</sup> <math>f'(0) = 1</math></li> <li>•<sup>4</sup> for the sketch</li> </ul> 
4	<ul style="list-style-type: none"> <li>•<sup>1</sup> for correct shape</li> <li>•<sup>2</sup> for annotation</li> <li>•<sup>3</sup> <math>f'(0) = 0</math></li> <li>•<sup>4</sup> <math>f'(2) = 0</math></li> <li>•<sup>5</sup> for correct shape</li> </ul> 
5	<ul style="list-style-type: none"> <li>•<sup>1</sup> know that there are <b>exactly two zeros</b></li> <li>•<sup>2</sup> <b>0 and 2</b></li> <li>•<sup>3</sup> any parabola with max t.p.</li> </ul> 
6	<ul style="list-style-type: none"> <li>•<sup>1</sup> <b>any one of</b> <math>f'(b) = 0</math>, <math>f'(d) = 0</math>, <math>f'(0) = 1</math></li> <li>•<sup>2</sup> <b>remaining two answers</b></li> <li>•<sup>3</sup> <b>shape of graph</b></li> <li>•<sup>4</sup> <b>annotation</b></li> </ul> 
7	<ul style="list-style-type: none"> <li>•<sup>1</sup> clear evidence of reflection in <math>y = 0</math></li> <li>•<sup>2</sup> clear evidence of translation <math>\begin{pmatrix} 0 \\ 2 \end{pmatrix}</math> subsequent to a reflection</li> <li>•<sup>3</sup> indication of passing through <math>(a,1)</math> and <math>(b,2)</math></li> <li>•<sup>4</sup> roots at <math>x = a</math> and <math>x = b</math></li> <li>•<sup>5</sup> parabolic shape with min. turning point between the roots and no other turning points</li> <li>•<sup>6</sup> <math>\left(0, \frac{1}{2}\right)</math></li> </ul>